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Solar and atmospheric neutrino oscillations from bilinear R-parity violation

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Abstract: We discuss general predictions for neutrino masses and mixing angles from R parity violation in the Minimal Supersymmetric Standard Model. If the soft supersymmetry breaking terms are flavor blind at short distance, then the leptonic analogue of the CKM matrix depends on only two real parameters, which are completely determined by fits to solar and atmospheric neutrino oscillations. Either the small angle MSW, large angle MSW, or "just-so" solutions to the solar neutrino problem are allowed, although the large angle MSW solution requires substantial fine-tuning. The latter two cases require significant $\nu_\mu \to \nu_e$ oscillations of atmospheric neutrinos. We present a model which could explain bilinear R parity violation as a consequence of spontaneous symmetry violation by a dynamical supersymmetry breaking sector. The decay length and branching ratios of the LSP are estimated.

Keywords: Solar and Atmospheric Neutrinos, Supersymmetric Standard Model, Neutrino Physics.

Contents

1. Evidence for neutrino oscillations and neutrino mass

Recently SuperKamiokande has reported an energy dependent and zenith angle dependent deficit of atmospheric muon neutrino events compared with theoretical expectations, which is strong evidence for oscillations of muon neutrinos [1]. The hypothesis of neutrino oscillations will soon be further tested through long baseline neutrino oscillation searches [2]. In addition, the solar electron neutrino flux observed by 5 different experiments is much less than expected [3], suggesting oscillations of electron neutrinos as well [4]. The simplest explanation for either anomaly is to modify the Standard Model to provide neutrinos with mass. The atmospheric neutrino data can be explained well by a large mixing angle for the muon and tau neutrinos and a neutrino mass squared difference between $\sim 5 \times 10^{-4}$ and $\sim 6 \times 10^{-3}$ eV^2. The solar neutrino flux deficits can be explained by the MSW [5] effect for either a large or a small electron neutrino mixing angle and a mass squared difference of order 10^{-5} eV^2 [6], or by "just-so" [7] vacuum oscillations with large electron neutrino mixing and a mass squared difference of $\sim 10^{-10}$ eV^2. If one relaxes the constraints coming from the standard solar model or from one or more of the solar neutrino experiments then other mass squared differences and mixing angles are possible [8].

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Oscillations amongst all three neutrinos can explain all solar and atmospheric data, provided one mass squared difference is in the atmospheric range and another in either the MSW or "just-so" solar ranges. Some models invoke oscillations to new neutrino states which are "sterile" under the weak interactions in order to also account for the $\nu_\mu \to \nu_e$ oscillation evidence reported by the LSND collaboration [9]. Several future experiments on solar and terrestrially produced neutrinos are expected to further constrain the possibilities [10].

In this note we will assume that there are only three neutrino states and will only explain the solar and atmospheric neutrino anomalies. We neglect the results of LSND as these have not been confirmed by any independent experiment.[1] We will show that a class of renormalizable supersymmetric theories with lepton number violation can account for these anomalies and examine the consequent predictions.

2. Bilinear R parity violation

Unlike the minimal Standard Model, the Minimal Supersymmetric Standard Model (MSSM) allows renormalizable baryon and lepton number violation. If neutrinos are massive Majorana particles, then lepton number is violated by nature, and there is no compelling reason to assume that lepton number violating terms are absent from the superpotential.[2]

In the MSSM, the most general renormalizable superpotential contains the terms

$$\frac{1}{2}\lambda_{ijk}\ell_i\ell_j\bar{e}_k + \lambda'_{ijk}\ell_i q_j \bar{d}_k + \frac{1}{2}\lambda''_{ijk}\bar{u}_i\bar{d}_j\bar{d}_k \,. \tag{2.1}$$

Another possible term is

$$\epsilon_i H_u \ell_i \,, \tag{2.2}$$

but this is conventionally eliminated by performing a redefinition of the H_d and lepton superfields

$$\begin{pmatrix} \ell_e \\ \ell_\mu \\ \ell_\tau \\ H_d \end{pmatrix} \to U \begin{pmatrix} \ell'_e \\ \ell'_\mu \\ \ell'_\tau \\ H'_d \end{pmatrix} , \tag{2.3}$$

where U is a unitary 4×4 matrix.

To avoid proton decay, all such terms are usually forbidden by imposing a discrete symmetry, known as R parity, under which the quark and lepton superfields change sign. However, avoiding proton decay only requires that either the baryon

[1] The KARMEN [11] and MINI-BooNE [12] experiments have the capability to check this result.

[2] It is however trivial to find discrete symmetries which forbid renormalizable lepton number violating terms but allow dimension five terms which give Majorana neutrino masses.

number violating terms or the lepton number violating terms vanish.[3] Supersymmetric models with either baryon or lepton number violation are known as R parity violating models [14, 15]. A special case of R parity violation, known as the bilinear R parity violating model (BRPVM), only contains the R parity violating terms which can be written in the form (2.2). This restricted model is more predictive and theoretically more attractive than more general models. The BRPVM arises naturally from a large class of theories in which R parity violation is spontaneous, such as many grand unified models [14], and, unlike general R parity violating model, has few potential phenomenological difficulties with flavor changing neutral currents and lepton flavor violation [16].

2.1 Superpotential and soft supersymmetry breaking terms

Although the BRPVM is defined by assuming the superpotential can be written in the form (2.2), it is more convenient to use the transformation (2.3) to write the superpotential in the canonical form of (2.1). The lepton number violating terms in the superpotential can then be written in terms of only 3 new parameters θ_i, $i = 1, 2, 3$ with

$$s_i, c_i \equiv \sin\theta_i, \cos\theta_i \tag{2.4}$$

and

$$s_1 \equiv \frac{\epsilon_e}{\sqrt{\epsilon_\mu^2 + \epsilon_e^2}}, \qquad s_2 \equiv \frac{\sqrt{\epsilon_\mu^2 + \epsilon_e^2}}{\sqrt{\epsilon_\tau^2 + \epsilon_\mu^2 + \epsilon_e^2}}, \qquad s_3 \equiv \frac{\sqrt{\epsilon_\tau^2 + \epsilon_\mu^2 + \epsilon_e^2}}{\sqrt{\mu^2 + \epsilon_\tau^2 + \epsilon_\mu^2 + \epsilon_e^2}}, \tag{2.5}$$

where μ is the coefficient of the superpotential term $H_u H_d$.[4] Neglecting the Yukawa couplings of the first two generations, we perform the transformation (2.3) on the MSSM superpotential with bilinear R parity violation using the matrix

$$U = \begin{pmatrix} c_3 & -c_2 s_3 & -c_1 s_2 s_3 & -s_1 s_2 s_3 \\ c_2 s_3 & c_3 & 0 & 0 \\ c_1 s_2 s_3 & 0 & c_3 & 0 \\ s_1 s_2 s_3 & 0 & 0 & c_3 \end{pmatrix}. \tag{2.6}$$

In this basis, the superpotential lepton number violating terms can be written

$$\lambda_b \bar{b} q_3 \Big\{ s_3[c_2 \ell_\tau + s_2(c_1 \ell_\mu + s_1 \ell_e)] \Big\} + \lambda_\tau \bar{\tau} \ell_\tau \Big\{ s_3[s_2(c_1 \ell_\mu + s_1 \ell_e)] \Big\}, \tag{2.7}$$

where $q_3 \equiv (t', b)_L$, t' is the electroweak partner of the b quark, and $\lambda_{b,\tau}$ are respectively the bottom and tau Yukawa couplings.

[3] This holds unless the model has a light non-leptonic fermion such as an axino or light gravitino [13].

[4] The superfield H_d has the same SM gauge quantum numbers as ℓ_i. We assume, however, that they are distinguished in the microscopic theory by R parity (or in general, some symmetry which contains R parity as a subgroup) and that the dynamical origin of ϵ_i and μ are necessarily different.

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In general there will also be the soft supersymmetry breaking lepton number violating terms in the scalar potential

$$B_{\epsilon_i}\epsilon_i H_u \tilde{\ell}_i + \tilde{m}^2_{H_d\ell_i} H_d^* \tilde{\ell}_i + A^{\lambda}_{ijk}\lambda_{ijk}\tilde{\bar{d}}_i\tilde{q}_j\tilde{\ell}_k + A^{\lambda'}_{ijk}\lambda'_{ijk}\tilde{\bar{e}}_i\tilde{\ell}_j\tilde{\ell}_k + h.c. \tag{2.8}$$

We assume lepton universality for the supersymmetry breaking terms, i.e. the messenger of supersymmetry breaking is blind to lepton flavor. This assumption allows us to simplify (2.8), in the same basis as (2.7), to

$$B_\epsilon \epsilon_i H_u \tilde{\ell}_i + \tilde{m}^2_{H_d\ell}\left(\frac{\epsilon_i}{\mu}\right) H_d^*\tilde{\ell}_i + A^d\lambda^d_i\left(\frac{\epsilon_j}{\mu}\right)\tilde{\bar{d}}_i\tilde{q}_i\tilde{\ell}_j + A^e\left(\lambda^e_i\left(\frac{\epsilon_j}{\mu}\right) - \lambda^e_j\left(\frac{\epsilon_i}{\mu}\right)\right)\tilde{\bar{e}}_i\tilde{\ell}_i\tilde{\ell}_j + \text{h.c.} \tag{2.9}$$

at the "messenger scale" Λ at which supersymmetry breaking is communicated, although at another scale all the terms in (2.8) will be generated by renormalization.

Lepton number violation allows neutrinos to obtain Majorana masses. One linear combination of neutrinos can gain a mass at tree level from the diagram in figure 1a and at one loop from figure 1b, while the loop diagrams of figure 2 can give the other neutrinos mass. The computations of sneutrino vevs contributing to these masses may be found in the appendix.

Generally this model produces an enormous hierarchy in the neutrino mass spectrum, with the ratio of the heaviest and second heaviest neutrino masses at least 10^3. In order to reduce this hierarchy, we can assume that the communication of supersymmetry breaking is also blind to the difference between a lepton and H_d, as would naturally occur with Gauge Mediated Supersymmetry Breaking [17] in which all soft supersymmetry breaking terms including those of (2.8) are generated by gauge interactions [18]. In such models, one can choose a basis where the bilinear lepton number violating soft supersymmetry breaking terms vanish at the messenger scale Λ, and loop effects are needed to generate a sneutrino vev.

2.2 The lepton mixing matrix

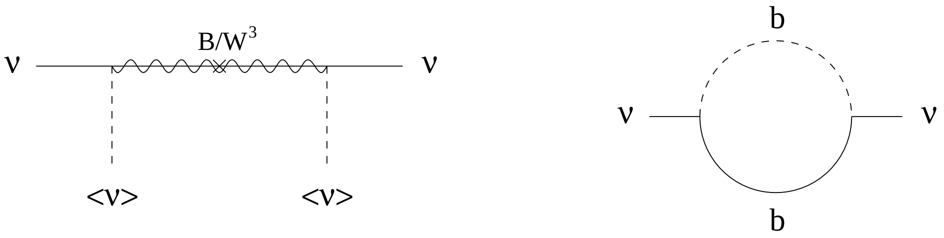
In the BRPVM, a very general argument shows that the lepton analogue of the CKM matrix can be predicted in terms of 2 mixing angles and is CP conserving.[5] This is most easily seen in the basis where all R parity violation is bilinear. Note that in the excellent approximation of neglecting the electron and muon Yukawa couplings, there is a U(3) flavor symmetry acting on the neutrinos which is broken by only two terms — the R parity violating term

$$\sum_{i=e,\mu,\tau} \epsilon_i H_u \ell_i \tag{2.10}$$

and the tau Yukawa coupling

$$\lambda_\tau \ell_\tau \bar{e}_\tau H_d \tag{2.11}$$

[5] Several recent papers [19] have analyzed neutrino masses in versions of the BRPVM, but have not noted this general prediction.



(a) (b)

Figure 1: Contributions to the largest neutrino masses at (a) tree level and (b) one loop.

(recall that we are assuming no additional flavor violation from supersymmetry breaking). The linear combination

$$(c_1\ell_e - s_1\ell_\mu) \tag{2.12}$$

is invariant under a chiral U(1) symmetry, which is broken only by tiny Yukawa couplings, and prevents this linear combination from gaining a mass. Thus one neutrino, which is purely a linear combination of ν_e and ν_μ, is always automatically very light compared with the other two. This argument is true for any of the possible mechanisms for generating the neutrino masses, provided only that the supersymmetry breaking terms respect lepton universality. The heaviest neutrino mass has no suppression factor due to the tau Yukawa coupling. This neutrino mass can result at tree level from sneutrino vevs, as well as from the one-loop graph in figure 1. Hence the heaviest neutrino, up to corrections involving λ_τ, is the linear combination

$$\sum_{i=e,\mu,\tau} \frac{\epsilon_i \ell_i}{\sqrt{\epsilon_\tau^2+\epsilon_\mu^2+\epsilon_e^2}} = c_2\nu_\tau + s_2(c_1\nu_\mu + s_1\nu_e)\,. \tag{2.13}$$

The mass of the second heaviest neutrino is proportional to both the R parity violating terms and the tau Yukawa coupling.

The preceding argument shows that in a basis where $j = 1,2,3$ labels neutrino mass eigenstates in ascending order of mass, the neutrino CKM matrix has the simple form

$$V^\nu = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_1 & s_1c_2 & s_1s_2 \\ -s_1 & c_1c_2 & c_1s_2 \\ 0 & -s_2 & c_2 \end{pmatrix}. \tag{2.14}$$

This is one of the main results of this paper.

For vacuum neutrino oscillations we predict

$$\begin{aligned} P_{e\leftrightarrow\mu} &= \sin^2 2\theta_1 \left[\left(c_2^2\sin^2\frac{\Delta m_{12}^2 L}{4E} + s_2^2\sin^2\frac{\Delta m_{13}^2 L}{4E}\right) - \frac{1}{4}\sin^2 2\theta_2 \sin^2\frac{\Delta m_{23}^2 L}{4E}\right], \\ P_{e\leftrightarrow\tau} &= s_1^2 \sin^2 2\theta_2 \sin^2\frac{\Delta m_{23}^2 L}{4E}, \\ P_{\mu\leftrightarrow\tau} &= c_1^2 \sin^2 2\theta_2 \sin^2\frac{\Delta m_{23}^2 L}{4E}. \end{aligned} \tag{2.15}$$

Note in the limit where Δm_{12}^2 can be neglected (relevant for atmospheric and terrestrial neutrino oscillations) that $P_{e\leftrightarrow\mu}$ simplifies to

$$P_{e\leftrightarrow\mu} \approx s_2^4 \sin^2 2\theta_1 \sin^2 \frac{\Delta m_{23}^2 L}{4E}\,, \tag{2.16}$$

while in the limit where Δm_{23}^2 is large enough so that many oscillation lengths are averaged over (relevant for "just-so" solar neutrino oscillations)

$$P_{e\not\to e} \approx \sin^2 2\theta_1 c_2^2 \sin^2 \frac{\Delta m_{12}^2 L}{4E} + 2 s_1^2 s_2^2 (c_2^2 + c_1^2 s_2^2)\,. \tag{2.17}$$

In general, matter effects significantly change these results for both solar and atmospheric neutrinos [5]. Several groups [20, 21, 22] have derived constraints on neutrino oscillation parameters, under the assumption that all 3 neutrino flavors mix. Using the results of Fogli et al. on atmospheric [21] and MSW solar [20] neutrino oscillations and the results of ref. [23] on "just-so" solar neutrino oscillations, we find three regions of parameter space which can give a good fit to the SuperKamiokande data and all the solar neutrino experiments:

1. The angle θ_2 is large ($\sin^2(2\theta_2) > 0.8$) and θ_1 is small ($s_1 \sim 0.07$). In this case the atmospheric neutrino oscillations are almost purely $\nu_\mu \leftrightarrow \nu_\tau$ and m_3 lies between $\sim 3 \times 10^{-2}$ and $\sim 10^{-1}$ eV. The solar neutrino problem is solved by the small angle MSW effect and $m_2 \sim 3 \times 10^{-3}$ eV.

2. Both θ_2 and θ_1 are large, with $s_1 \sim 0.7$ and $s_2 \sim 0.8$, and the solar neutrino problem is solved by the large angle MSW effect. In this case $m_3 \sim 3 \times 10^{-2}$ eV and $m_2 \sim 10^{-2}$ eV. Since the ratio m_2/m_3 is always rather small unless there are finely tuned accidental cancelations, this solution seems rather unlikely.

3. A small region of parameter space is within the 90% confidence limits for both atmospheric neutrinos and "just-so" solar neutrinos with $s_1 \sim 0.5$, $s_2 \sim 0.8$, $m_2 \sim 10^{-5}$ eV and $m_3 \sim 3 \times 10^{-2}$ eV. This large ratio m_3/m_2 is easily produced in a generic model of supersymmetry breaking.

The long baseline reactor search for electron neutrino disappearance, KamLAND [24], will definitively distinguish these three possibilities, being sensitive to Δm^2 larger than $\sim 10^{-5}\,\mathrm{eV}^2$. In the small angle MSW case there is a negligible possibility of ν_e disappearance, the large angle case will give (averaging over both oscillation lengths) $P_{e\not\to e} \sim 0.6$ (note that an average disappearance probability greater than 1/2 is possible when the electron neutrino is a linear combination of more than two mass eigenstates) and in the just-so case, where only one Δm^2 is large enough to be observable in a terrestrial experiment, $P_{e\not\to e} \sim 0.3$.

Note that in all cases, the ratio m_2/m_3 must lie between $\sim .3$ and $\sim 3 \times 10^{-4}$. As we will discuss in the next section, m_2 is always proportional to a loop factor suppressed by powers of λ_τ, and so models which can give a large enough value for this ratio are quite constrained.

2.3 Predictions for neutrino mass

The possible dominant contributions[6] to the heaviest neutrino mass m_3 are from the tree diagram, figure 1a, and the one loop diagram, figure 1b, while m_2 can only arise from the one loop diagrams of figure 2, and is always suppressed by λ_τ. Typically the tree diagram is much larger than any loop diagrams, and gives a contribution to m_3 of

$$m_3 = \left(\frac{g^2}{4M_2} + \frac{g'^2}{4M_1}\right) \langle\tilde{\nu}\rangle^2 \,, \tag{2.18}$$

where g, g' are Standard Model gauge couplings, $M_{1,2}$ are Majorana gaugino masses and we have neglected mixing in the neutralino mass matrix (reasonable for $m_z \ll M_{1,2}$ or $m_z \ll \mu$). However, the sneutrino vevs are naturally suppressed relative the the parameters ϵ_i, when universal soft supersymmetry breaking terms are generated at a low scale. Note that small sneutrino vevs are necessary if m_2/m_3 is large enough to allow the MSW solution to the solar neutrino problem and are most natural when $\langle H_u\rangle/\langle H_d\rangle \equiv \tan\beta$ is large, the soft supersymmetry breaking terms are completely universal, and the messenger scale is low, as in a gauge mediated scenario. In the appendix we compute the sneutrino vevs and find that a large value for $\tan\beta$, a messenger scale of 50 TeV, and an accidental cancelation of order 10% between two independent contributions to the sneutrino vev will allow the loop contributions to be within a factor of five of the tree contribution to m_3. Thus we expect the tree contribution to dominate. The contribution of the one loop diagram to m_3 is

$$m_3 = \frac{\lambda_b^4 s_3^2}{16\pi^2 \tan\beta} \frac{\mu \langle H_u\rangle^2 \log(\tilde{m}_{b_1}^2/\tilde{m}_{b_2}^2)}{\tilde{m}_{b_1}^2 - \tilde{m}_{b_2}^2} \,, \tag{2.19}$$

where

$$\tilde{m}_{b_{1,2}}^2 = \frac{1}{2}\left[\left(\tilde{m}_{b_L}^2 + \tilde{m}_{b_R}^2\right) \pm \sqrt{\left(\tilde{m}_{b_L}^2 - \tilde{m}_{b_R}^2\right)^2 + 4\lambda_b^2 \mu^2 \langle H_u\rangle^2}\,\right]. \tag{2.20}$$

There are several possible contributions[7] to m_2 — which one is largest depends on the size of s_3. For large s_3, the diagram of figure 2a dominates, giving

$$m_2 = \frac{\lambda_\tau^4 s_3^2 s_2^2 c_2^2}{16\pi^2 \tan\beta} \frac{\mu \langle H_u\rangle^2 \log(\tilde{m}_{\tau_1}^2/\tilde{m}_{\tau_2}^2)}{\tilde{m}_{\tau_1}^2 - \tilde{m}_{\tau_2}^2} \,, \tag{2.21}$$

where $\tilde{m}_{\tau_{1,2}}$ are defined analogously to $\tilde{m}_{b_{1,2}}$.

[6] We compute contributions to neutrino masses in the trilinear basis of (2.7). The loop diagrams we compute have effectively been taken into account in [25] (see also [26]), but in the basis where the neutrino vev vanishes. For a discussion of a basis independent description of R parity violation, see [27].

[7] These contributions are not strictly to m_2 but to a linear combination of m_2 and m_3. For $s_2^2 \sim 1/2$ (approximately true in all cases), there is an additional contribution to the second heaviest neutrino's mass of order m_2^2/m_3, and to the mixing of order m_2/m_3. These contributions are only non-negligible in the MSW scenario where $m_2/m_3 \sim 1/10 - 1/30$.

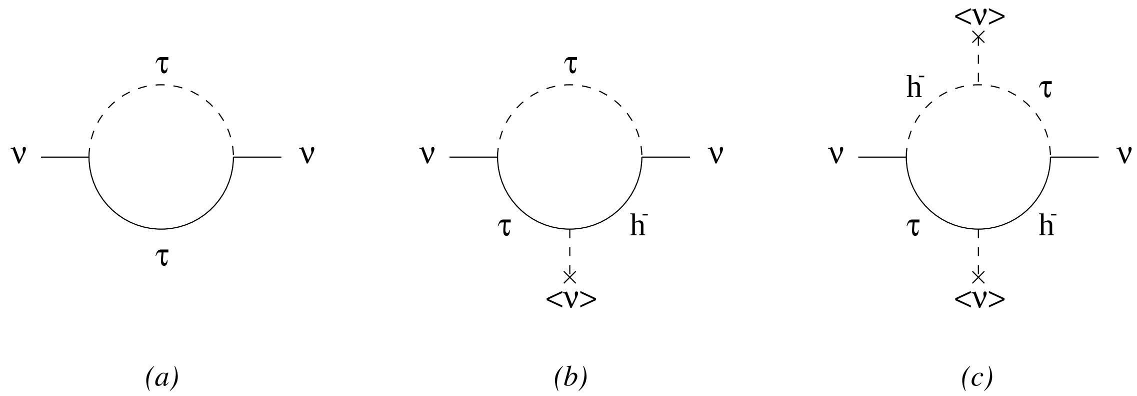


Figure 2: One loop contributions to the second neutrino mass.

For moderate s_3 and large $\tan\beta$, the dominant contribution to m_2 would come from the diagram of figure 2b, giving

$$m_2 = \frac{\lambda_\tau^4 s_3 s_2^2 c_2^2}{16\pi^2} \frac{\langle\tilde{\nu}\rangle\langle H_u\rangle}{\mu} f\left(\frac{\tilde{m}_{\tau 1}^2}{\mu^2}, \frac{\tilde{m}_{\tau 2}^2}{\mu^2}\right), \tag{2.22}$$

where

$$f(x,y) = \frac{x\log(x) - y\log(y) - xy\log(x/y)}{(1-x)(1-y)(y-x)}. \tag{2.23}$$

When this diagram dominates the ratio m_2/m_3 is always too small for an MSW solution to the solar neutrino problem, however a "just-so" solution is possible for large $\tan\beta$.

Finally, for small s_3, the dominant contribution to m_2 comes from the diagram of figure 2c, which gives

$$m_2 = \frac{\lambda_\tau^4}{16\pi^2} \frac{A_\tau\langle\tilde{\nu}\rangle^2}{\mu^2} \left[f\left(\frac{m_{h^-}^2}{\mu^2}, \frac{\tilde{m}_{\tau 1}^2}{\mu^2}\right) s_{\theta_\tau}^2 + f\left(\frac{m_{h^-}^2}{\mu^2}, \frac{\tilde{m}_{\tau 2}^2}{\mu^2}\right) c_{\theta_\tau}^2 \right], \tag{2.24}$$

where θ_τ is the mixing angle between the left and right handed tau sleptons. Note that this contribution is very small unless λ_τ is large, which requires large $\tan\beta$. Large $\tan\beta$ is possible only if either h_d is heavy or the soft supersymmetry breaking bilinear $h_u h_d$ term is small. Known mechanisms for suppressing such a bilinear term suppress the trilinear A_τ term as well. Thus for this case, for any known predictive theory, m_2/m_3 is too small to allow a simultaneous fit to atmospheric and solar neutrino data.

Note that the ratio m_2/m_3 is proportional to λ_τ^4 in all cases. Thus obtaining a large enough value of m_2/m_3 to account for both solar and atmospheric neutrino anomalies always requires large (~ 50) $\tan\beta$.

One additional way in which it may be possible to produce a viable hierarchy of neutrino masses is through gravitational effects. In the case of a large vev compared to $\epsilon \equiv \sqrt{\epsilon_\tau^2 + \epsilon_\mu^2 + \epsilon_e^2}$, m_2 is too small even for the "just-so" solution. However, m_2

Superfield	q	u^c	d^c	ℓ	e^c	H_u	H_d
$\mathrm{U}(1)_R$	$\frac{1}{2}+a$	$\frac{1}{2}-a$	$\frac{1}{2}-a$	$\frac{1}{2}+b$	$\frac{1}{2}-b$	$+1$	$+1$

Table 1: Charge assignments for a global $\mathrm{U}(1)_R$ symmetry

may receive an interesting contribution from a non-renormalizable operator in the superpotential:

$$W \supset \lambda^{\nu}_{ij} \frac{H_u \ell_i H_u \ell_j}{M_p} , \tag{2.25}$$

where $M_p \simeq 2 \times 10^{18}$ is the reduced Planck mass. With λ^{ν} of order one, this produces a neutrino mass $m_\nu \sim \langle H^u \rangle^2 / M_p \sim 10^{-5}\,\mathrm{eV}$, precisely what is needed for "just-so" solar neutrino oscillations. However, there is no definite prediction for the leptonic CKM mixing matrix in this case.

3. Models of bilinear R parity violation

Supersymmetric models which generate only bilinear R parity violation are quite natural. In this section we provide an existence proof of such a model.

One way to produce such terms is via a spontaneously broken R symmetry. For example, a $\mathrm{U}(1)_R$ symmetry exists in the MSSM with the charge assignments of table 1.[8]

For $a = -b/3$, all renormalizable R parity violating terms have R charge $(3/2+b)$.

Now, at a scale M, we introduce a new sector of particles which does not couple to the observable sector via contact or gauge interactions, and in which the $\mathrm{U}(1)_R$ symmetry is broken spontaneously. Thus the R symmetry breaking can only be communicated via gravitational effects. Assume there exists a gauge invariant operator $\mathcal{O}$ of mass dimension n and R charge $(1/2 - b)$. If $\mathcal{O}$ has an expectation value $\langle \mathcal{O} \rangle = M^n$, then the following term in the superpotential of the low energy effective theory,[9]

$$\frac{\mathcal{O}}{M_p^{n-1}} \ell H_u , \tag{3.1}$$

produces a bilinear R parity violating term with coupling $\epsilon \sim M^n / M_p^{n-1}$. Trilinear R parity violating terms would in general also be produced with couplings of order M^n / M_p^n, and therefore would be greatly suppressed compared to the bilinear term.

[8] A continuous $\mathrm{U}(1)_R$ symmetry has $\mathrm{SU}(2)\times\mathrm{U}(1)$ anomalies. However, R parity is an anomaly free discrete subgroup. The anomalies could always be canceled by another sector, such as a messenger sector.

[9] In general, there will be a number of gauge invariant operators which fit this description. Here we consider the operator with the lowest mass dimension n. Those operators with mass dimension greater than n will have contributions suppressed by powers of the Planck mass.

An interesting possibility is to have the operator $\mathcal{O}$ come from the supersymmetry breaking sector. However, in order to produce a neutrino spectrum from the R parity violating term which is consistent with both solar and atmospheric anomalies, we have seen that the parameter ϵ must not be much smaller than the sneutrino vev $\langle\tilde{\nu}\rangle$. This naturally occurs if all lepton violating soft breaking terms in (2.8) have coefficients no larger than $B\epsilon$, with B of order the weak scale. However, if $\mathcal{O}$ is in the supersymmetry breaking sector, it would generally also have an F-term expectation value leading to a scalar bilinear of order

$$\frac{FM^{n-1}}{M_p^{n-1}}\tilde{\ell}H_u\,. \tag{3.2}$$

Generically, the coefficient of this term is much too large. For example, if $F/M \sim 10^5$ GeV (as is typical in models of gauge mediated supersymmetry breaking) then $\langle\tilde{\nu}\rangle/\epsilon \sim 10^2$. Therefore, if R symmetry breaking and supersymmetry breaking come from the same sector, then the heaviest neutrino's mass m_3 comes from R parity violation, while m_2 comes from Planck slop, as described at the end of section 2.3.

4. Predictions for the decays of the (N)LSP

In the BRPVM, with parameters chosen to fit the atmospheric and solar neutrino data, most effects of the lepton number violating couplings are too small too be observed. The main evidence for R parity violation will only come when supersymmetry is discovered. The Lightest Supersymmetric Particle (LSP) decays via the R parity violating couplings. In some models, the LSP is an ultralight gravitino, however the lifetime for the decay of the next lightest supersymmetric particle (NLSP) into the gravitino is very long, and so the NLSP will primarily decay via R parity violating couplings.

How the (N)LSP decays depends on whether neutrinos oscillate according to the MSW solution or the "just-so" solution. The MSW case requires a small sneutrino vev compared to the R parity violating parameters, $\langle\tilde{\nu}\rangle \lesssim \epsilon/100$, in order for (2.18) to give a mass which satisfies atmospheric neutrino data. Decays which depend on couplings proportional to $s_3 \simeq \epsilon/\mu$ dominate. The "just-so" solution requires $\langle\tilde{\nu}\rangle \gtrsim \epsilon$ to give a large mass ratio m_3/m_2. In this case, the dominant decays of the (N)LSP are due primarily to the mixing of leptons with neutralinos and charginos (except in decays to top quarks as mentioned below). With the R parity violating couplings fixed to explain neutrino masses and mixing, we can make definite predictions for the dominant decay mode of the (N)LSP, which depends only on which superpartner it is and its mass. In some cases, we can also predict the lifetime.

For instance, in the MSW case, if the (N)LSP is the lightest tau slepton $\tilde{\tau}_1$ (and $\tilde{m}_{\tau_1} < m_{\text{top}}$) it will decay into a charged lepton and neutrino with a lifetime between 10^{-15} and 10^{-16} seconds. The $\tilde{\tau}_1$ could decay somewhat less often into $b\bar{c}$ quark jets.

The branching ratios of $\tau\nu$, $\mu\nu$, $e\nu$ and $b\bar{c}$ are proportional to $(\lambda_\tau s_2)^2$, $(\lambda_\tau s_2 c_1 s_{\theta_\tau})^2$, $(\lambda_\tau s_2 s_1 s_{\theta_\tau})^2$ and $3(\lambda_b V_{cb} c_2 c_{\theta_\tau})^2$ respectively. In the case of the "just-so" solution, the $\tilde{\tau}_1$ decays nearly exclusively into $\ell\nu$ (mostly $\tau\nu$) with a lifetime of about 10^{-13} seconds. However, if $\tilde{m}_{\tau_1} \to b\bar{t}$ is kinematically allowed, it will be the dominant decay mode in the MSW scenario and may be comparable to the leading decay mode in the "just-so" case.

Another likely possibility for the (N)LSP is the lightest neutralino N_1, which has a small neutrino component. In the MSW scenario, the N_1 will decay predominantly into $b\bar{b}$ quark jets and a neutrino (or $b\bar{t}\bar{\ell}$ if energetically allowed). The lifetime in this case is proportional to four powers of the mass $\tilde{m}_\ell$ of the exchanged slepton when $\tilde{m}_\ell^2 \gg M_{N_1}^2$. In the case of the "just-so" solution, the dominant decay will be a charged lepton and a $W^\pm$, with an interesting decay length of 0.1-10 mm [28]. The branching ratios for N_1 decays into e, μ and τ are respectively proportional to $s_2^2 s_1^2$, $s_2^2 c_1^2$, and c_2^2. Also possible are decays into a neutrino and a Z, with a branching fraction approaching 25% when the N_1 is very heavy and mostly Bino [28]. There can also be a significant branching ratio for the mode $h^0\nu$ if kinematically allowed.

5. Summary

The Bilinear R Parity Violating Model is a well motivated and predictive theory of neutrino masses. The BRPVM can account for both the atmospheric and solar neutrino anomalies, and restricts the form of the leptonic charged current mixing matrix, which can depend only on two independent angles. A large electron neutrino mixing angle solution to the solar neutrino problem is possible if and only if the atmospheric oscillations involve a significant ν_e component. The hierarchy between neutrino masses is generically very large, and the MSW solution to the solar neutrino problem is reasonably natural only if $\tan\beta$ is large, soft supersymmetry breaking terms for the leptons and Higgses are universal, and the scale of transmission of supersymmetry breaking is low, suggesting gauge mediated generation of the soft supersymmetry breaking terms. With the typical neutrino mass ratio, vacuum oscillations can solve the solar neutrino problem. Definitive tests of this theory will be provided by KamLAND which can distinguish the MSW from "just-so" solutions, and by observation of the pattern of (N)LSP decays.

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A. On sneutrino vev diagrams

Some suppression of sneutrino vevs is necessary if the heaviest neutrino is not to be too much heavier than the next heaviest. Here we examine the sneutrino vevs assuming that at some scale Λ there is no supersymmetry breaking mixing term between Higgs scalars and sneutrinos. Motivated by gauge mediated models, we also assume the A terms vanish at Λ. We assume that Λ is small enough so that we do not need to use the renormalization group, since when Λ is large so will be the sneutrino vev.

At one loop, there are contributions to $\tilde{\nu}$-H_d mixing:

$$\tilde{m}^2_{H_d\ell_i} = \tilde{m}^2_{H_d\ell}\frac{\epsilon_i}{\mu} = \frac{N_c\lambda_b^2\epsilon_i}{16\pi^2\mu}\left(\tilde{m}^2_{b_1}\log\frac{\Lambda^2}{\tilde{m}^2_{b_1}} + \tilde{m}^2_{b_2}\log\frac{\Lambda^2}{\tilde{m}^2_{b_2}}\right). \tag{A.1}$$

There is also a one loop contribution to the A terms at scale k:

$$A_b(k^2) = \left(\frac{4}{3}\right)\frac{\alpha_s}{2\pi}\lambda_b s_3 M_3\int_0^1 \log\frac{\Lambda^2}{x\left((1-x)k^2+M_3^2\right)}dx\,, \tag{A.2}$$

where M_3 is the gluino mass. The graphs with winos/binos are somewhat smaller, for example the wino contribution will be suppressed compared to the above by only $\sim\ \left(\frac{3}{4}\alpha_2 M_2\right)/\left(\frac{4}{3}\alpha_3 M_3\right)\sim 1/10$. The one loop contribution to $\tilde{\nu}$-H_u mixing is

$$B_{\epsilon_i}\epsilon_i = B_\epsilon\epsilon_i = \left(\frac{4}{3}\right)\frac{\alpha_s}{2\pi}\frac{\lambda_b^2}{16\pi^2}\frac{\epsilon_i}{\mu}N_c\mu M_3\left[-\frac{1}{2}\left(\log\frac{\Lambda^2}{\tilde{m}_b^2}\right)^2 - \log\frac{\Lambda^2}{\tilde{m}_b^2}\right], \tag{A.3}$$

where we've assumed small chargino mixing. For simplicity, the result shown is to zeroth order in $(\tilde{m}^2_{b_2}-\tilde{m}^2_{b_1})$.

The Higgs vevs give us terms in the potential which are linear in the sneutrinos with a coefficient

$$C_{\tilde{\nu}} = \left(B_\epsilon\mu + \frac{\tilde{m}^2_{H_d\ell}}{\tan\beta}\right)s_3\langle H_u\rangle, \tag{A.4}$$

and thus give non-zero sneutrino vevs. For degenerate sneutrino masses, the vev is $\langle\tilde{\nu}_3\rangle = C_{\tilde{\nu}}/\tilde{m}^2_\nu$, where $\tilde{\nu}_3$ is the partner of ν_3, the heaviest neutrino in our basis. The sneutrinos are not exactly degenerate since $\tilde{m}^2_{\nu_\tau}$ will get an additional contribution from one-loop diagrams proportional to λ_τ^2, but this effect is small.

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